Chapter 7
Case Studies in Materials Design

7.0 Introduction

7.1 Applications Using Metals and Alloys

7.1.1 Compressed Air Tank

(Adapted from Lewis, and Ashby)

You are asked to design a vessel to contain compressed air. Economics and weight are obviously issues to consider, but the primary consideration is safety; *i.e.*, the tank must not rupture. Specifically, the tank must meet three design criteria:

1. The maximum stress in the vessel must be below the yield strength of the material used.
2. The vessel must not fail by fast fracture.
3. The vessel must not fail by fatigue.

The vessel is subjected to an internal pressure from the compressed air, which we shall designate as p. The internal pressure is uniformly distributed over the internal surfaces of the vessel, giving rise to both circumferential stress, σ_H, also known as *hoop stress*, and longitudinal stress, σ_L (see Figure 7.1). We will examine each of these stresses independently before we begin the material selection process. In our development, we will make the following assumptions:

- The radial stresses in the cylinder wall are negligible
- There are no longitudinal stays in the cylinder

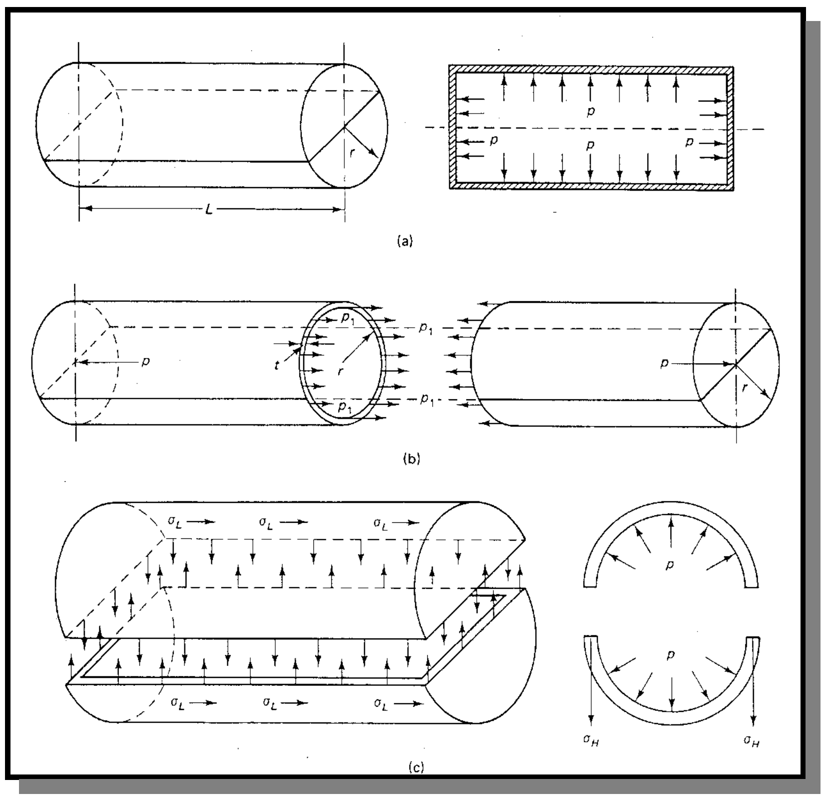

Figure 7.1 Stresses in a thin cylinder subjected to an internal pressure, *p*: (a) cylindrical shell under internal fluid pressure; (b) longitudinal stress development; (c) hoop stress development.

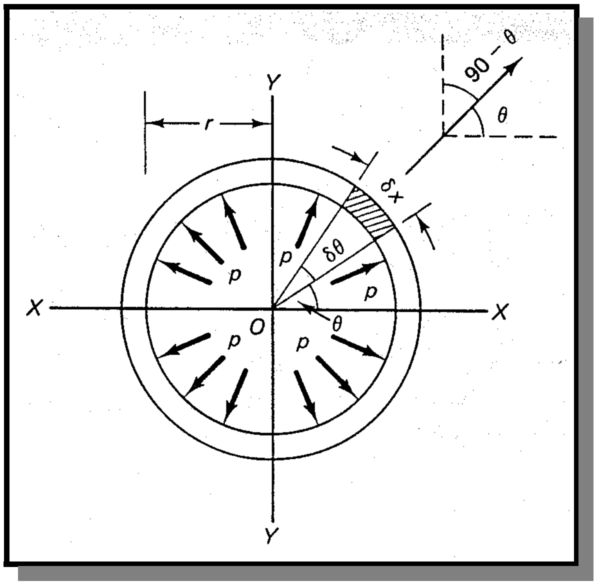

Figure 7.2 Element of the wall of a thin cylinder subjected to internal pressure *p*.

- The stresses are uniformly distributed throughout the section wall.

The hoop stress can be obtained by considering an elemental portion of the cylinder wall, δx, subtending an angle $\delta\theta$, at the center and at an angle θ with XX (see Figure 7.2). The internal radius of the cylinder is r and the length of the unit is L. Since the pressure of the fluid always acts perpendicular to the surface of contact, the total pressure, P, normal to the elemental section is given by:

$$P = p \times \text{(area of elemental section)} = p\delta x L = pr\delta\theta L \quad \textbf{(7.1.1)}$$

The vertical component of the pressure is, δP_v:

$$\delta P_v = pr\delta\theta L \sin\theta \quad \textbf{(7.1.2)}$$

The total upward pressure on the semicircular portion of the cylindrical shell above the diametral plane XX is obtained by integrating δP_v over the entire angle, θ:

$$P_v = \int_0^{\pi} prL\sin\theta\,\delta\theta = 2prL \quad \textbf{(7.1.3)}$$

Similarly, the total downward pressure on the semicircular portion of the cylindrical shell below the diametral plane XX is also $2prL$. These two equal and opposite pressures act to burst the cylinder longitudinally at the plane XX. The resisting force comes from the hoop stress. Thus

$$2prL = 2\sigma_H tL \quad \textbf{(7.1.4)}$$

where t is the thickness of the wall. We can then solve equation 7.1.4 for the hoop stress

$$\sigma_H = pr/t \quad \textbf{(7.1.5)}$$

(This is the hoop stress for thin-walled vessels, *i.e.*, $t<r/4$. For thick-walled vessels, see Ashby Fig. A11.) The longitudinal stress can be obtained from a similar shell balance (see Figure 7.1):

$$\sigma_L = pr/2t \quad \textbf{(7.1.6)}$$

By comparing equations 7.1.5 and 7.1.6, we see that the maximum stress on the vessel is given by the hoop stress, and is equal to pr/t. Let us now turn our attention to the design criteria.

Criterion 1: *The maximum stress in the vessel must be below the yield strength of the material used.*

To avoid yield of the tank

$$\sigma_H = pr/t < \sigma_y \qquad \textbf{(7.1.7)}$$

where σ_y is the yield stress of the material used. The higher the value of σ_y , the higher the hoop stress that can be tolerated in the vessel.

If we consult the strength vs. density diagram of Ashby (Fig. 7.3), we see that the classes of materials with the highest strengths are engineering ceramics, engineering composites, and engineering alloys. Of the engineering alloys, the titanium, steel and nickel alloys provide the highest ranges of strengths. With these material classes in mind, let us continue to the other criteria.

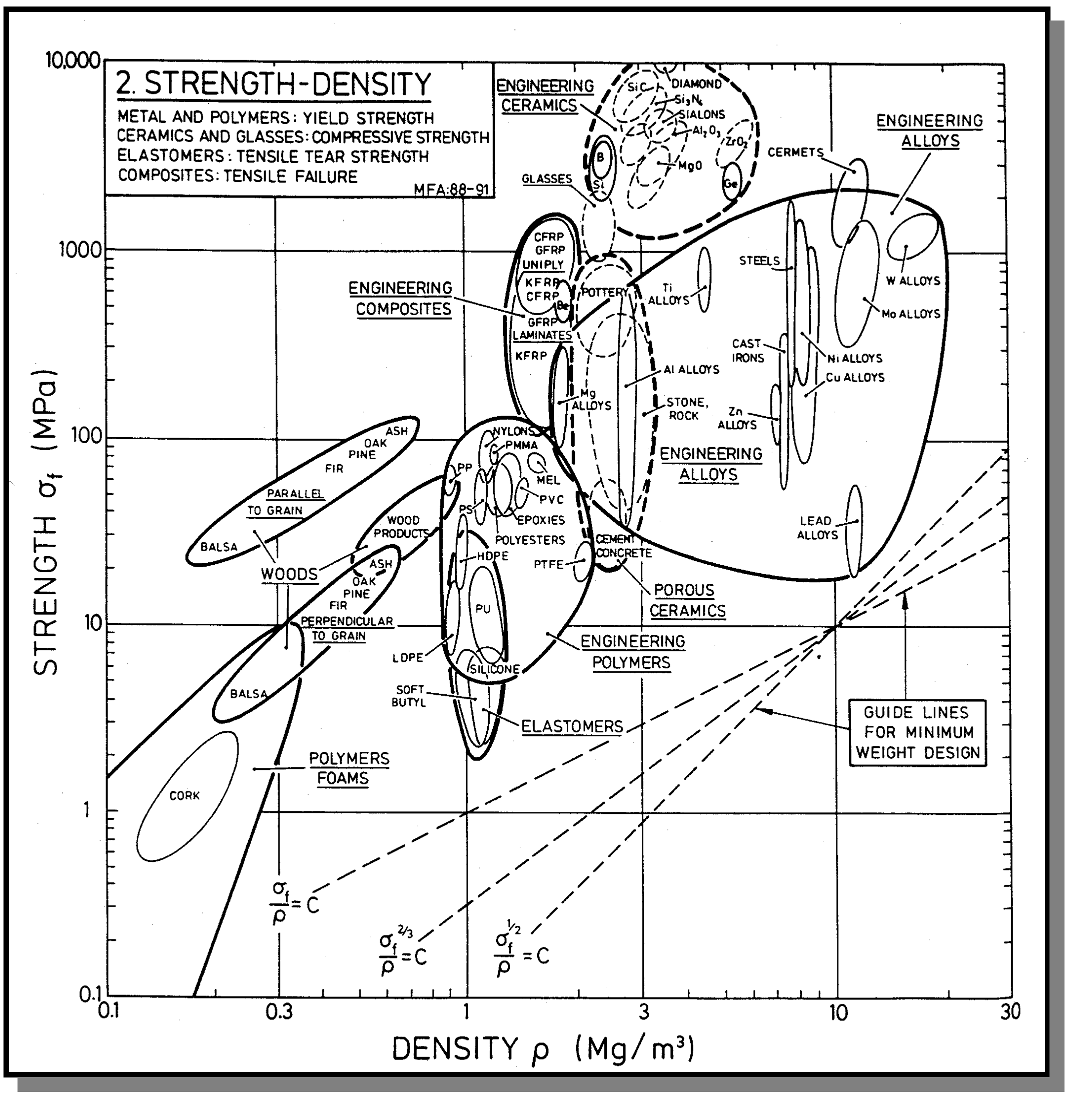

Figure 7.3 Strength vs. density for materials classes.

Criterion 2: *The vessel must not fail by fast fracture.*

Recall that the opening mode stress intensity factor for the case of a component containing a single edge crack in tension is given by:

$$K_I = 1.12\sigma\sqrt{\pi a} \quad \textbf{(7.1.8)}$$

where σ is the applied stress and a is the length of the edge crack in the vessel. (See Ashby , Fig. A10 for formulae for internal cracks or two cracks.) To avoid failure by fast fracture, the following condition must be met:

$$K_{IC} \geq 1.12\sigma_H\sqrt{\pi a} \quad \textbf{(7.1.9)}$$

where σ_H is the maximum applied stress, a_{cr} is the critical flaw size, and K_{IC} is the fracture toughness of the vessel material. From equation 7.1.9,

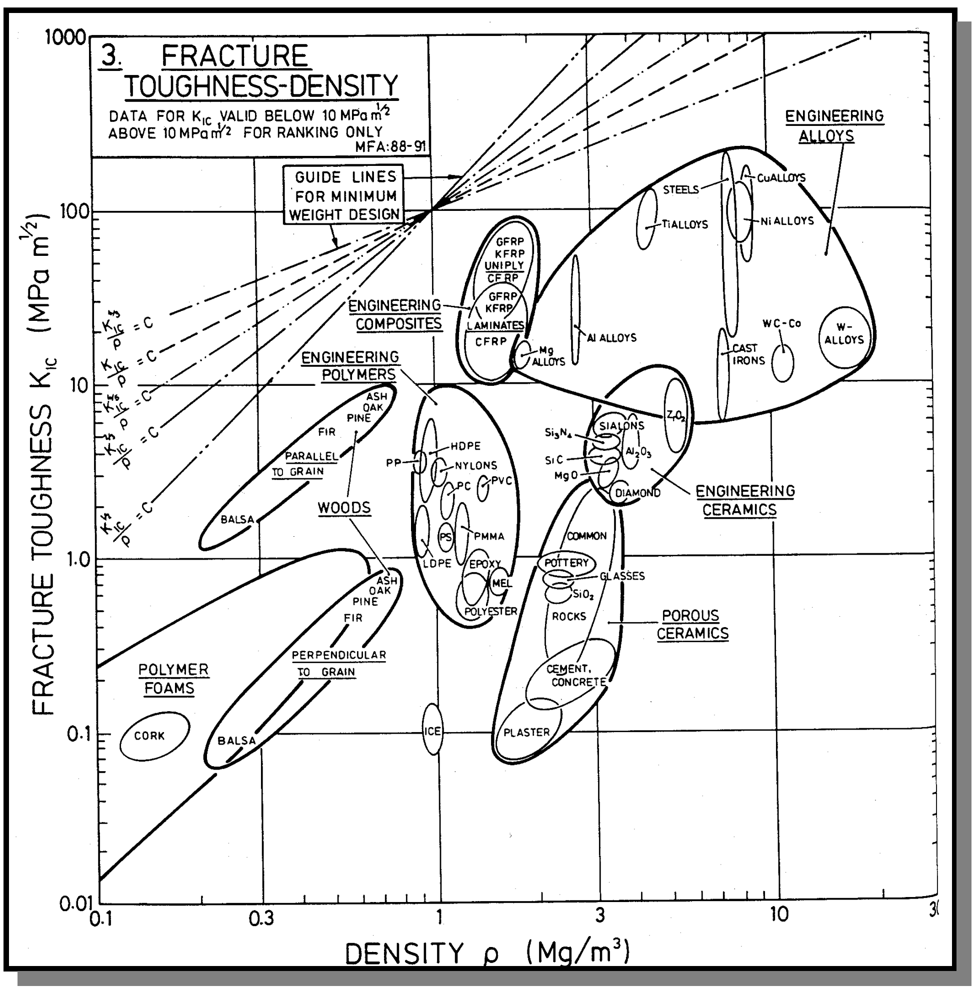

Figure 7.4 Fracture toughness vs. density for materials classes.

$$a_{cr} = \frac{\left[K_{IC}/1.12\sigma_H\right]^2}{\pi} \tag{7.1.10}$$

That is, a_{cr} is directly proportional to $(K_{IC}/\sigma_y)^2$ since σ_H is a fraction of σ_y. Thus, the larger the value of a_{cr}, the more attractive is the material, since cracks can be easily detected without the use of sophisticated equipment. The Ashby plot of fracture toughness vs. density (Figure 7.4) indicates that of the three classes of materials selected with Criterion 1, only the engineering composites and engineering alloys provide suitable possibilities for Criterion 2. Again, of the alloys, titanium, steel, nickel and copper alloys are the best here.

The real power of the Ashby diagrams comes when we realize that we can combine Figures 7.3 And 7.4 to yield one, more useful, diagram (Figure 7.5), namely a plot of fracture toughness vs. strength. This plot shows unequivocally that the steel, nickel and titanium alloys are the best classes of materials to select for this application. We will use Criterion 3 to narrow this field even further.

Criterion 3: *The vessel must not fail by fatigue.*

Assuming that the fatigue crack growth law in the vessel material is describable by the Paris-Erdogan Law, we see that

$$da/dN = B(\Delta K_t)^n \tag{7.1.11}$$

where da/dN is the fatigue crack growth rate, ΔK_t is the range of stress intensity factor, and B and n are material constants. The number of cycles to failure, N_f, of the component is given by

$$N_f = \frac{2[a_i^{1-(n/2)} - a_{cr}^{1-(n/2)}]}{(n-2)B\alpha^n(\Delta\sigma)^n\pi^{n/2}} \tag{7.1.12}$$

when $n \neq 2$. In the present case, α=1.12 (see equation 7.1.8). So, for the crack to grow from an initial size a_i to a critical size a_{cr}, the rate of growth is dependent upon K_{IC}. We also want small n, and B to get a large number of cycles to failure, N_f.

Material Selection

Though there are many possibilities of the engineering alloys, let us consider three common alloys from different classes: a steel, an aluminum alloy, and a titanium alloy. The three alloys and their appropriate design properties are listed in Table 7.1. The values that are the most favorable in each category are listed in bold typeface. On the basis of Criterion 1, the best material is maraging steel, but from the viewpoints of Criteria 2 and 3, the titanium alloy is obviously superior. Cost is an additional factor that could influence the final selection.

Table 7.1 Comparison of Three Materials for Compressed Air Tank.

Material	σ_y (Mpa)	K_{IC} (MPa·m$^{1/2}$)	B	n	a_{cr} (mm)	N_f
18% Ni maraging steel	**1340**	100	1.32 x 10^{-10}	**2.32**	5.65	1955
Al-Zn-Mg alloy	490	25	6.34 x 10^{-11}	3.14	2.64	2205
Ti-6Al-4V	830	**120**	**9.60 x 10^{-13}**	3.80	**21.21**	**4513**

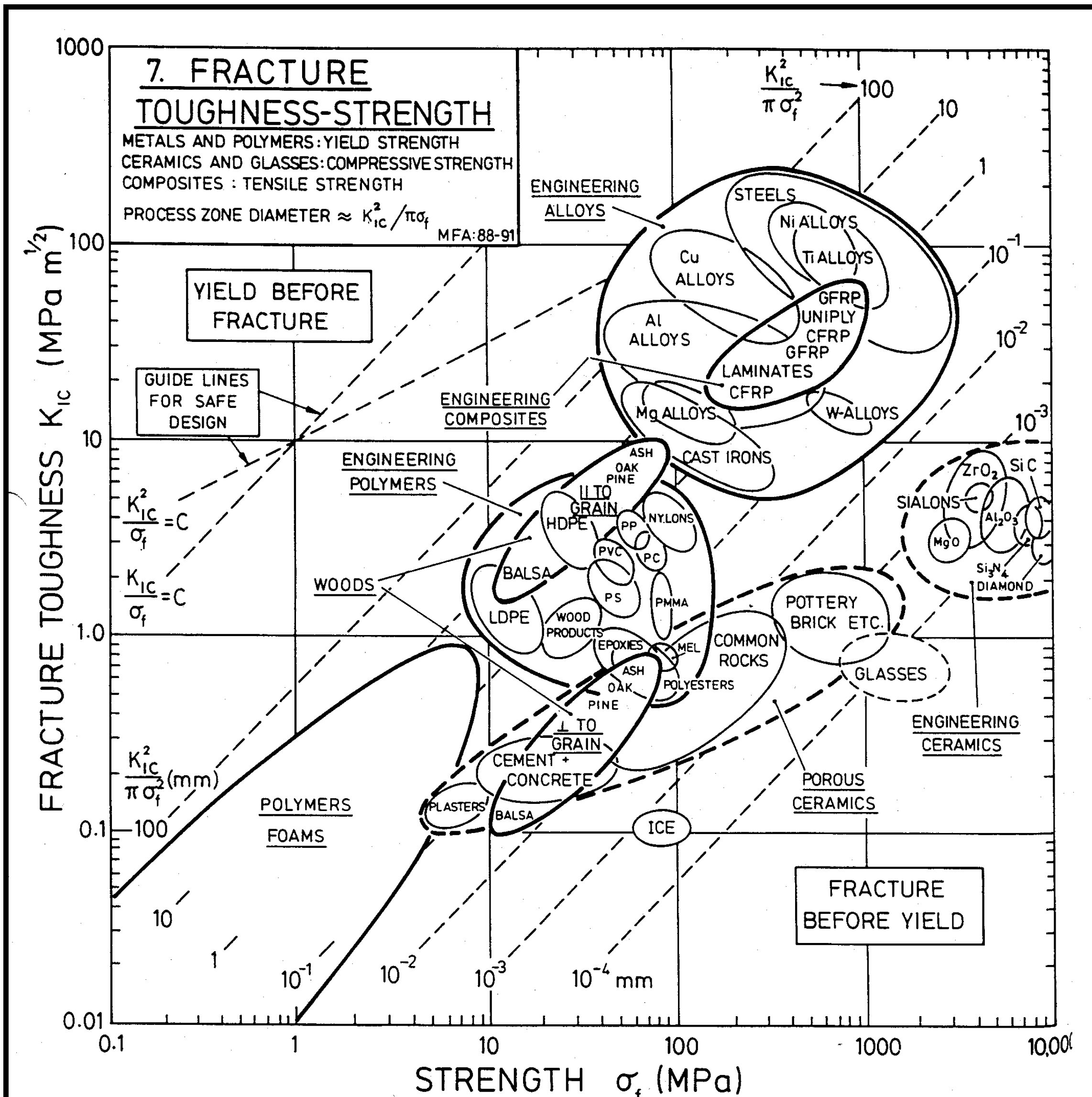

Figure 7.5 Fracture toughness vs. strength for materials classes.